

2004

e

Design and development of e-learning material to foster
problem solving ability in mathematics

2312002009

G.

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Presently, an objective on draft of the Education M

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		66
2	1 1	66
1 1	(1).....	66
1 1	(2).....	70
	1 1	73
1 1	(1).....	73
1 1	(2).....	75
		78
2	1 1	85
1 1	(1).....	85
1 1	(2).....	87
		90
		91

URL

<http://www.et.soft.iwate-pu.ac.jp/~g231a009/>

研究の背景

研究の動機

1)

2)

3)

(1998)

p.10 f ω 3 74

システムの設計

システム構成

3.1

3

問題解決プロセスの目標設定

2.2

3.1

3.1

原因の解決法の設定

3.4

3.5

3.4

			2,3 _____
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3.5

			2,3

3.4

3.5

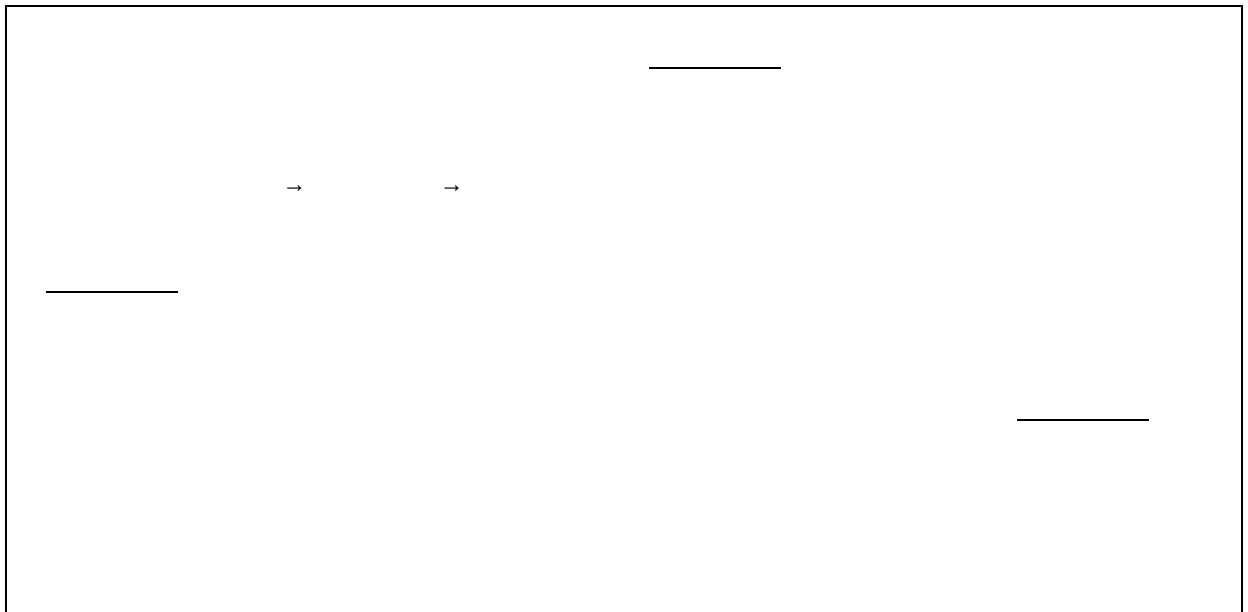
2.2

事前テストフェーズ

「二次関数」と「個数の処理」の共通項目

3.6

3.6



3.2

3.2

3.1

3.7 1

2

3-1

3-2

4-1

4-2

3.7 1 _____

	○ ○ × ×	
	○ ○ × ×	
3-1		
3-2		
3-2	○ ○	
	○ ○	
3-2		
3-2		
3-2	○ ○ × ×	
	○ ○ × ×	
3-2		
4-1		
4-1		
4-1	○ ○	
4-1		
4-1		
4-1	○ ○ × ×	
	○ ○ × ×	
4-1		
4-2		
4-2		

4-2	○ ○	
4-2		
4-2	○ ○ × ×	
4-2	○ ○ × ×	
4-2		

「二次関数」と「個数の処理」の相違項目

2

5

3.8

3.8

	$y=x^2-(m+2)x+4$ m	x^2 4 3 3 2
	$y=x^2-2(m-4)x+4$ m	x 3 5 3 2
	$y=x^2-(m-1)x+1$ m	x^2 5 3 3 2
	$y=x^2+2(k-4)x+8$ k	x 3 4 3 2

3.7



3.9

1

1

1

1

3.4

3.5

3-1

3.8

3.9

3.7

1	m $2 > m, m > 4$ $2 < m < 4$	27 5
1	$\circ \circ$ $\times \times$	$\circ \circ$ $\times \times$

	○ ○ × ×	× × ○ ○
	× × ○ ○	
3-1	m<-6,m>2	1440 2

3.7

3.4

3.5

3.7

2

3.10

3.10 2

3.7

3-2

4-1

4-2

3.11

13

$$\begin{array}{ccccccc} & & 3 & & & & \\ & & 3 & & 1 & & \\ & & & & & & 1 & \bullet \\ \circ & & & & & & & \end{array}$$

$$\begin{array}{ccccccc} & & 1 & & & & \\ \bullet & \bullet & \bullet & \bullet & \circ & \circ & \circ \end{array}$$

$$\begin{array}{ccccccccccc} & & & & 5! & & & & 1 & & \\ & & 3 & & & & 3! & & & & \\ 5! \times 3! = 5 & 4 & 3 & 2 & 1 \times 3 & 2 & 1 = 720 \end{array}$$

$$\begin{array}{ccccccc} & & 2 & & & & \\ & 2 & & 1 & & & 2 \\ & 2 & & & & & \\ \bullet & \bullet & \bullet & \bullet & \circ \end{array}$$

「二次関数」の追加問題における相違部分

$$2 \rightarrow 2 \rightarrow 2$$

3.14 17

3.14

	$y=x^2-(m+2)x+4$ m	$x^2+x+1>m(x+1)$ m
	$y=x^2-2(m-4)x+4$ m	$2(x^2-5x+15)<m(7-2x)$ m
	$y=x^2-(m-1)x+1$ m	$2x^2+4x+6>m(2x+1)$ m
	$y=x^2+2(k-4)x+8$ k	$k^2+4k-4<m(1-x)$ m

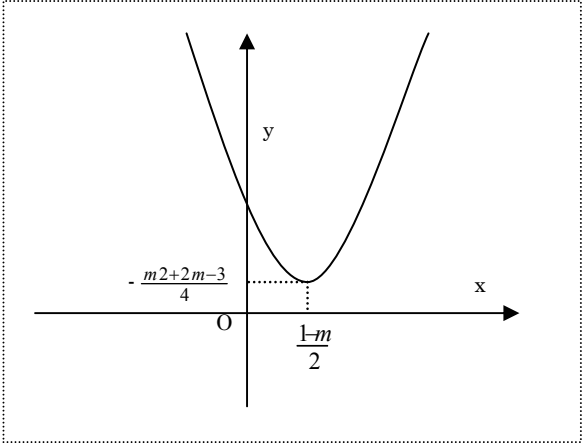
3.15

3.7

3-1	$m<-6, m>2$	$-3<m<1$
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3.16 3-2

4-1

$x^2+x+1>m(x+1)$ $x^2+(1-m)x+1-m>0$ $y=x^2+(1-m)x+1-m$ $=\{x+(1-m)+\frac{(1-m)^2}{4}\}-\frac{(1-m)^2}{4}+1-m$ $=(x+\frac{1-m}{2})^2-\frac{m^2+2m-3}{4}$ $(-\frac{1-m}{2}, -\frac{m^2+2m-3}{4})$  $-\frac{m^2+2m-3}{4}>0$ $m^2+2m-3<0$ $(m+3)(m-1)<0$ $m<-3, m>1$	$x^2+x+1>m(x+1)$ $x^2+x+1-mx-m>0$ $x^2+(1-m)x+1-m>0$ $x^2+(1-m)x+1-m=0$ $D=(1-m)^2-4(1-m)$ $=m^2+2m-3$ $=(m+3)(m-1)$ <0 $(m+3)(m-1)<0$ m $-3<m<1$
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3.17 4-2

$x^2+x+1 \quad m(x+1) \quad 1 \quad m$ $\frac{1}{2}(x^2+2x+5)>-m(x+1) \quad m$ $x^2+x+1<m(x+1) \quad m$ $\frac{1}{2}(x^2+2x+5) \quad -m(x+1) \quad m$	
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学習フェーズ

3

「二次関数」と「個数の処理」の共通項目

3.18

3.18



4

3.19

3.19

3.22

3.23

3.24

3.25

3.26

3.4 3.5
3.27 32

3.27

2,3	
	2,3

3.28

	3.19
	3.19

3.29

3.30

	3.21
	3.21

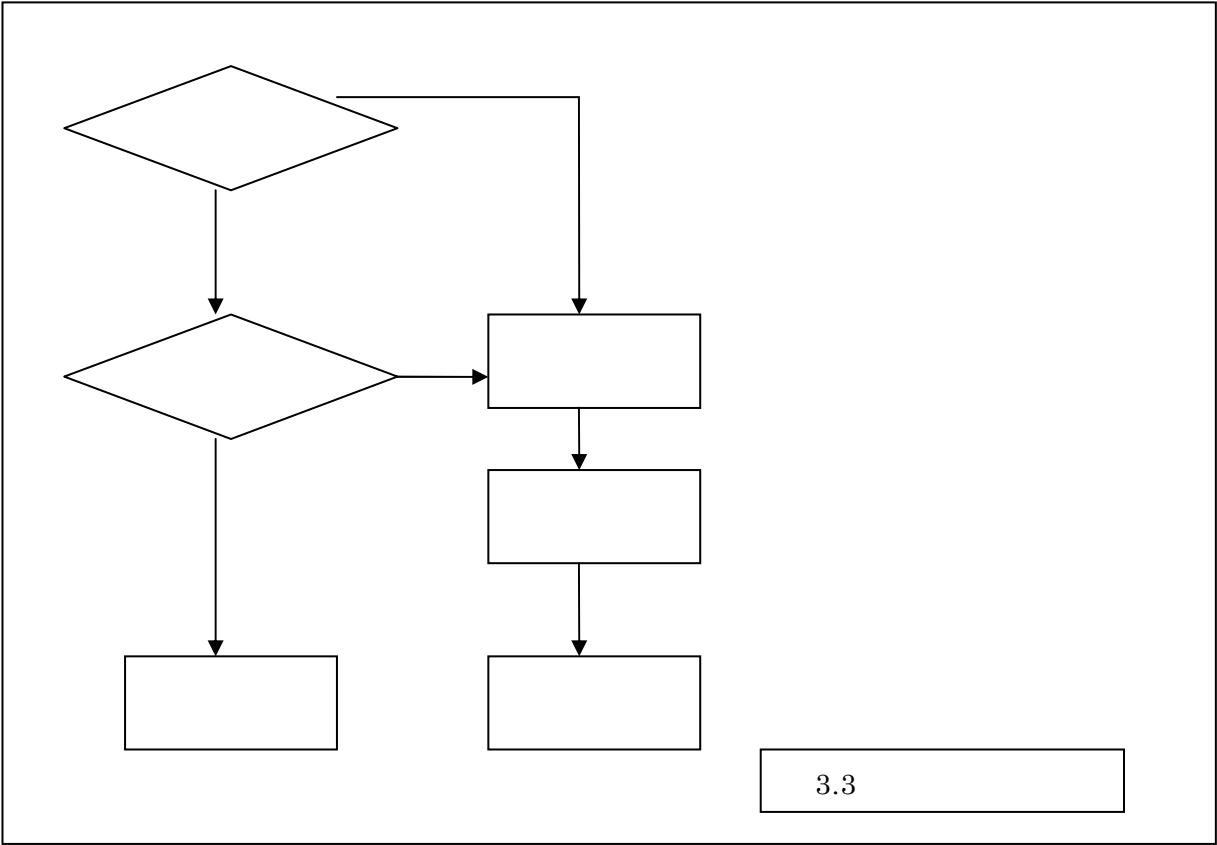
3.31

	3.19
	3.19

3.32

	3.19
	3.19

3.3



3.7

3.3

3.7 2

3.3

3.7 2

3.3

3.7

2

2

3.3

3.7 2

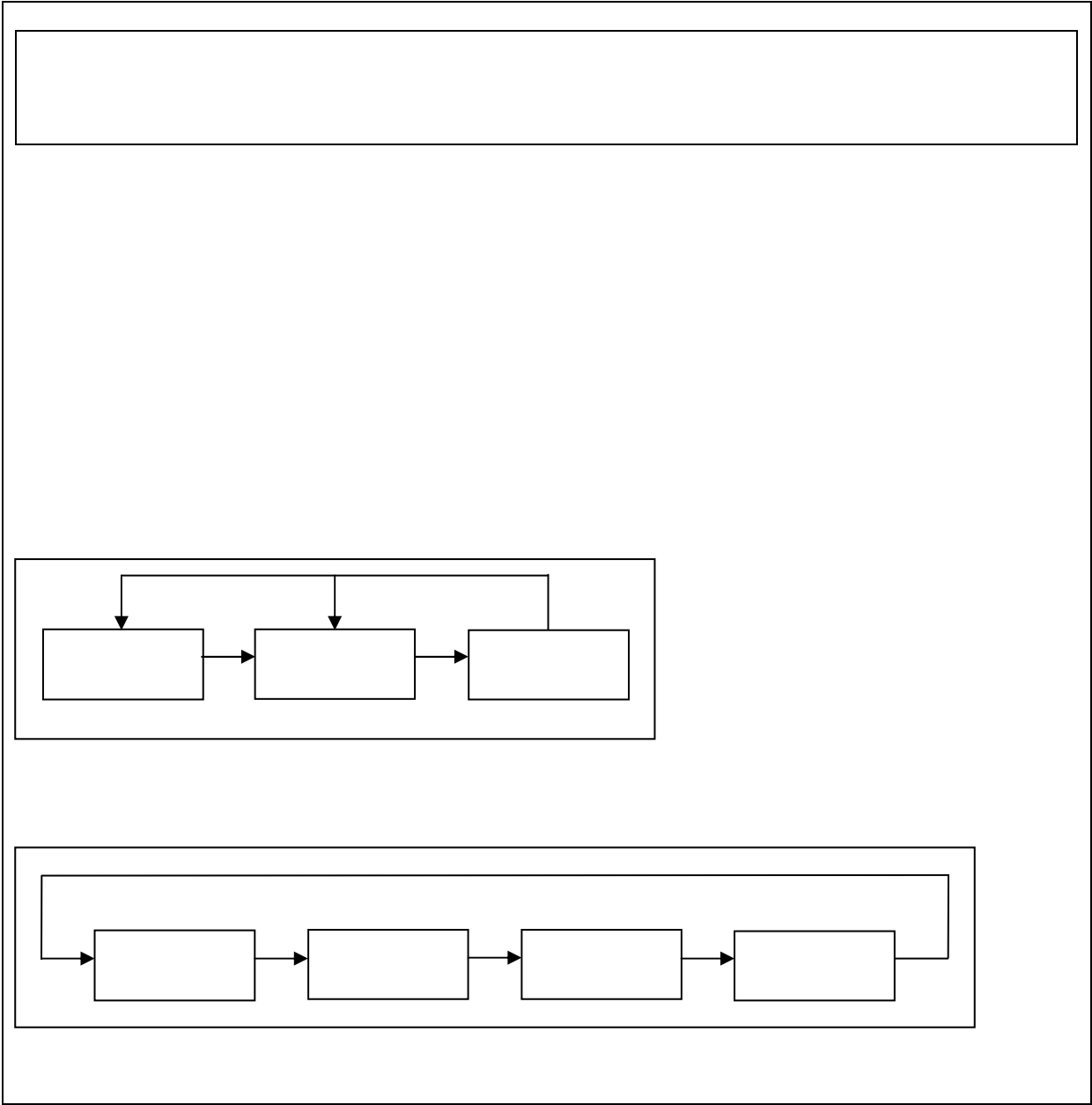
2

3-1

3

3.33

3.33



3.34

m<-6 , m>2

1440

3 2

「二次関数」と「個数の処理」の相違項目

3.4 3.5

3.37 2

3.38 2

3.4 3.5 2

3.39 44

3.39

	2 $y=ax^2+bx+c \rightarrow y=a(x-p)^2+q$ $ax+b=0 \rightarrow ax=-b$ $ax=-b \rightarrow x=-\frac{b}{a}$ $b-ax^3+cx-dx^2 \rightarrow -ax^3-dx^2+cs+b$ $x^2-4x+3=0 \rightarrow (x-3)(x-1)=0$ $(x+2)(x-1)=0 \rightarrow x^2+x-2=0$

_____	$ax^2+bx+c=0$ b^2-4ac Discrimination $D=b^2-4ac$

3.40

_____	A A 10 A A A 10 6 4 10 4=6
_____	× × ○ ○

3.41

_____	3.31
_____	3.31
_____	3.31

3.42

_____	3.32
_____	3.32
_____	3.32

3.43

_____	$y=x^2+4x+4$ _____ $y=x^2+2x+1$ _____ $y=x^3+4x+4$ $y=x^2+3x+1$

$$y = x^2 - (m+2)x + 4$$

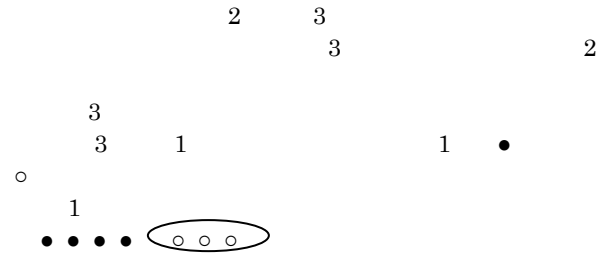
$$x^2 - (m+2)x + 4 = 0$$

$$D = (m+2)^2 - 16$$

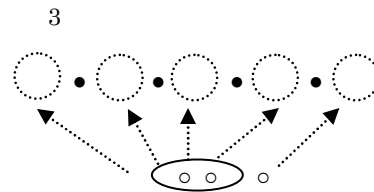
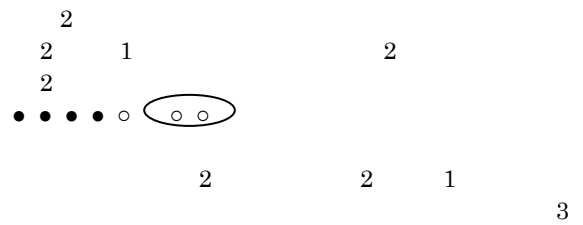
$$= m^2 + 4m - 12$$

$$x^2 - (m+2)x + 4 = 0$$

$$D > 0$$



$$5! \times 3! = 5 \times 4 \times 3 \times 2 \times 1 \times 3 \times 2 \times 1 = 720$$



$$5P2 \times 3P2 \times 5! = 5 \times 4 \times 3 \times 2 \times 4 \times 3 \times 2 \times 1 = 2880$$

$$720 + 2880 = 3600$$

$$5440 - 3600 = 1840$$

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3.47

-6>m , m>2	

$$y=x^2-(m+2)x+4 \tag{1}$$

$$=(x^2-(m+2)x+\frac{(m+2)^2}{4})-\frac{(m+2)^2}{4}+4 \tag{2}$$

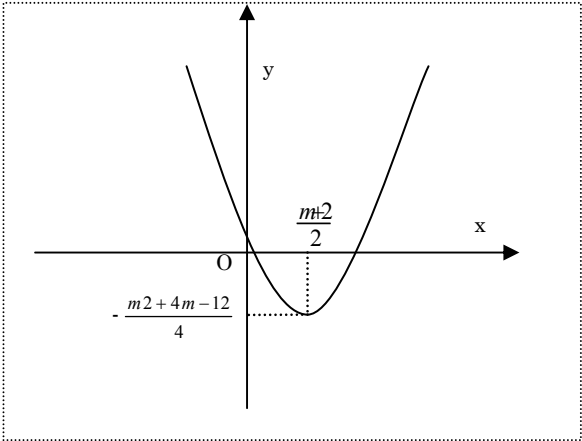
$$=(x-\frac{m+2}{2})^2-\frac{m^2+4m+4+16}{4} \tag{3}$$

$$=(x-\frac{m+2}{2})^2-\frac{m^2+4m+20}{4} \tag{4}$$

$$(\frac{m+2}{2}, -$$

3.48

3.49

21 $y = x^2 - (m+2)x + 4$$\rightarrow y = \left(x - \frac{m+2}{2}\right)^2 - \frac{m^2+4m-12}{4}$$\left(\frac{m+2}{2}, -\frac{m^2+4m-12}{4}\right)$  $-\frac{m^2+4m+12}{4} < 0$$x \quad 2$ $y = x^2 - (m+2)x + 4$$= \left(x - \frac{m+2}{2}\right)^2 - \frac{m^2+4m-12}{4}$$= \left(x - \frac{m+2}{2}\right)^2 - \frac{(m+6)(m-2)}{4}$	12 • ○ • • • • ○ ○ ○ ○ 2 2 ○ • ○ • ○ • ○ • ○ ○ ○ ○ 2 5 3 P3 4! P3 × 4! = 1440 7! = 5040 4 2 3 2 3 3 2
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$$=m^2+4m-12$$

$$D > 0$$

$$5440 - 3600 = 1440$$

4-1

4-2

3.11 13

3.53

3.9

3-1

3-1	$m < -1, m > 3$	3 1440

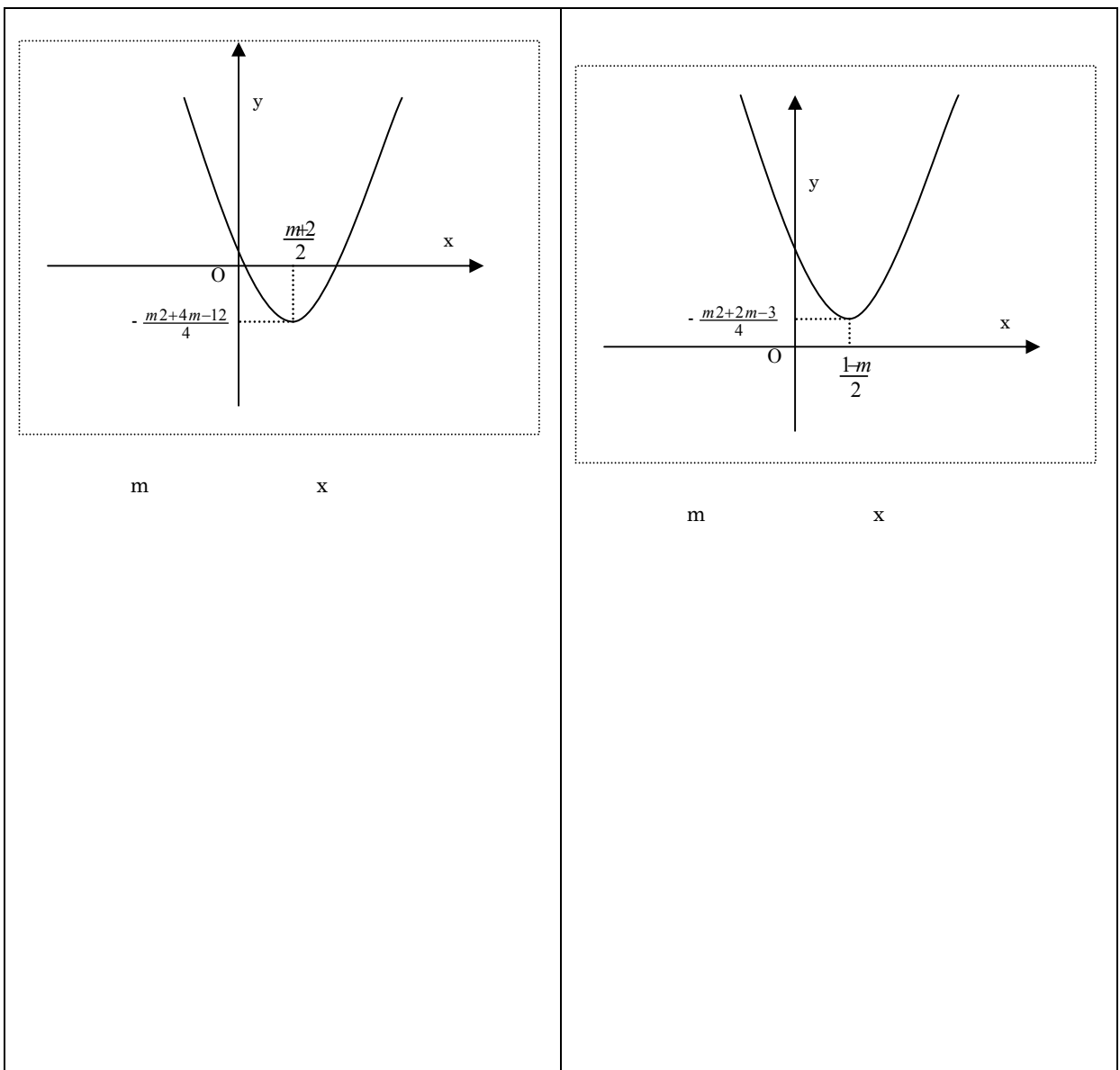
「二次関数」の追加問題における相違部分

3.54 59

3.14 15

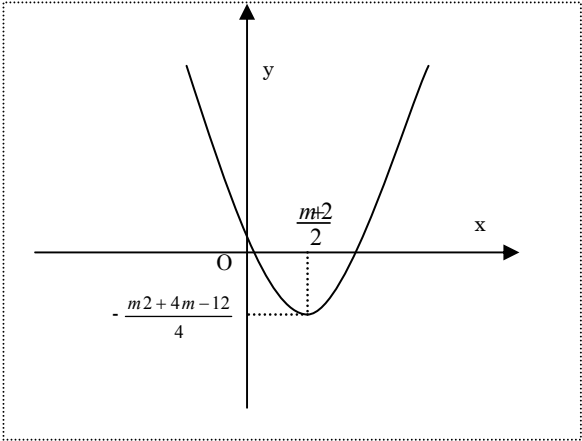
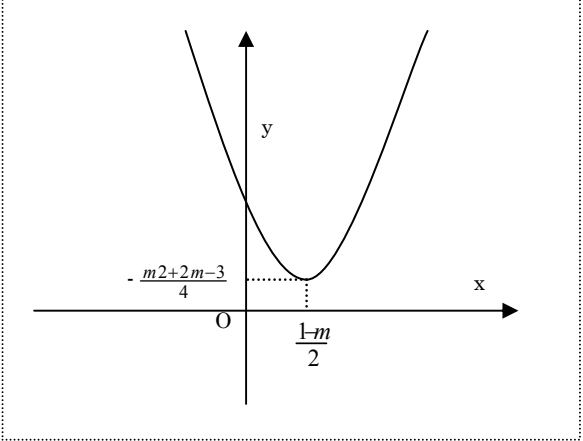
3.54

m $m > 2$ $0 \leq m \leq 5$ $y = x^2 - (m+2)x + 4$ $\rightarrow y = x^2 - (m+2)x + 4$ $\rightarrow x = m$ $\rightarrow$ $y = x^2 - (m+2)x + 4$ $\rightarrow y = \left(x - \frac{m+2}{2}\right)^2 - \frac{m^2+4m-12}{4}$ $\left(\frac{m+2}{2}, -\frac{m^2+4m-12}{4}\right)$	m $m > 2$ $0 \leq m \leq 5$ $x^2 + x + 1 > m(x+1)$ $\rightarrow x^2 + (1-m)x + 1 - m > 0$ x $y = x^2 + (1-m)x + 1 - m$ $\rightarrow$ $x = m$ $\rightarrow$ $y = x^2 + (1-m)x + 1 - m$ $\rightarrow y = \left(x + \frac{1-m}{2}\right)^2 - \frac{m^2+2m-3}{4}$ $\left(-\frac{1-m}{2}, -\frac{m^2+2m-3}{4}\right)$



3.55

$y = x^2 - (m+2)x + 4$	$y = x^2 + (1-m)x + 1-m$

$\rightarrow y = \left(x - \frac{m+2}{2}\right)^2 - \frac{m^2+4m-12}{4}$ $\left(\frac{m+2}{2}, -\frac{m^2+4m-12}{4}\right)$  $-\frac{m^2+4m+12}{4} < 0 \quad x^2$ $y = x^2 - (m+2)x + 4$ $= \left(x - \frac{m+2}{2}\right)^2 - \frac{m^2+4m-12}{4}$ $= \left(x - \frac{m+2}{2}\right)^2 - \frac{(m+6)(m-2)}{4}$ $y = x^2 - (m+2)x + 4$ $x^2 - (m+2)x + 4 = 0$ $D = (m+2)^2 - 16$ $= m^2 + 4m - 12$ $x^2 \quad D > 0$	$\rightarrow y = \left(x + \frac{1-m}{2}\right)^2 - \frac{m^2+2m-3}{4}$ $\left(-\frac{1-m}{2}, -\frac{m^2+2m-3}{4}\right)$  $-\frac{m^2+4m+12}{4} > 0 \quad y > 0$ $y = x^2 + (1-m)x + 1-m$ $y = x^2 - (1-m)x + 1-m$ $= \left(x - \frac{1-m}{2}\right)^2 - \frac{m^2+2m-3}{4}$ $= \left(x - \frac{1-m}{2}\right)^2 - \frac{(m+3)(m-1)}{4}$ $y = x^2 - (1-m)x + 1-m$ $x^2 - (1-m)x + 1-m = 0$ $D = (1-m)^2 - 4 \quad (1-m)$ $= m^2 + 2m - 3$ $x \quad D < 0$
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3.56

-6>m , m>2 $y=x^2-(m+2)x+4 \quad (1)$ $=(x^2-(m+2)x+\frac{(m+2)^2}{4})-\frac{(m+2)^2}{4}+4 \quad (2)$ $=(x-\frac{m+2}{2})^2-\frac{m^2+4m+4+16}{4} \quad (3)$ $=(x-\frac{m+2}{2})^2-\frac{m^2+4m+20}{4} \quad (4)$ $y \quad (\frac{m+2}{2}, -\frac{m^2+4m+20}{4})$ $\begin{matrix} (2) & (3) \\ (3) & -\frac{m^2+4m+4-16}{4} \\ & -\frac{m^2+4m+4+16}{4} \end{matrix}$ -6>m , m>2	-3<m<1 $y=x^2-(1-m)x+1-m \quad (1)$ $=(x^2-(1-m)x+\frac{(1-m)^2}{4})-\frac{(1-m)^2}{4}+1-m \quad (2)$ $=(x-\frac{1-m}{2})^2-\frac{m^2-2m+1+4-4m}{4} \quad (3)$ $=(x-\frac{1-m}{2})^2-\frac{m^2-6m+5}{4} \quad (4)$ $y \quad (\frac{1-m}{2}, -\frac{m^2-6m+5}{4})$ $\begin{matrix} (2) & (3) \\ (3) & -\frac{m^2-2m+1-4+4m}{4} \\ & -\frac{m^2-2m+1+4-4m}{4} \end{matrix}$ -3<m<1

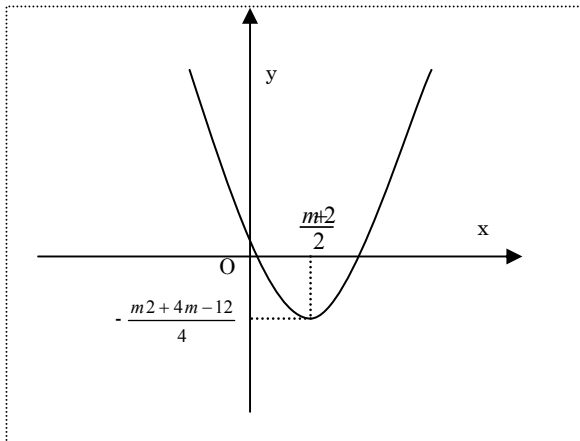
3.57

2 1	2 1

$$y = x^2 - (m+2)x + 4$$

$$\rightarrow y = \left(x - \frac{m+2}{2}\right)^2 - \frac{m^2+4m-12}{4}$$

$$\left(\frac{m+2}{2}, -\frac{m^2+4m-12}{4}\right)$$



$$-\frac{m^2+4m+12}{4} < 0 \quad x^2$$

$$y = x^2 - (m+2)x + 4$$

$$= \left(x - \frac{m+2}{2}\right)^2 - \frac{m^2+4m-12}{4}$$

$$= \left(x - \frac{m+2}{2}\right)^2 - \frac{(m+6)(m-2)}{4}$$

$$y = x^2 - (m+2)x + 4$$

$$x^2 - (m+2)x + 4 = 0$$

$$D = (m+2)^2 - 16$$

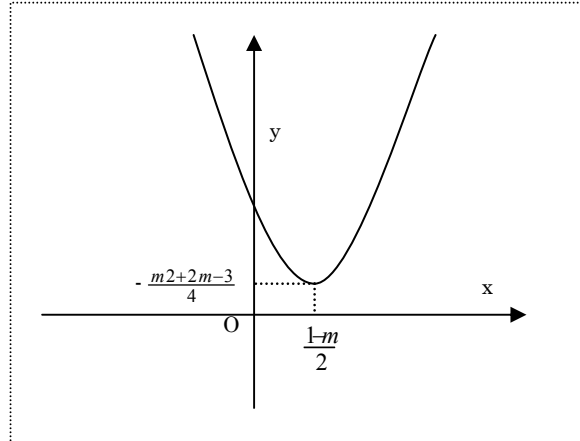
$$= m^2 + 4m - 12$$

$$x^2 \quad D > 0$$

$$y = x^2 + (1-m)x + 1-m$$

$$\rightarrow y = \left(x + \frac{1-m}{2}\right)^2 - \frac{m^2+2m-3}{4}$$

$$\left(-\frac{1-m}{2}, -\frac{m^2+2m-3}{4}\right)$$



$$-\frac{m^2+4m+12}{4} > 0$$

$$y = x^2 + (1-m)x + 1-m \quad y > 0$$

$$y = x^2 - (1-m)x + 1-m$$

$$= \left(x - \frac{1-m}{2}\right)^2 - \frac{m^2+2m-3}{4}$$

$$= \left(x - \frac{1-m}{2}\right)^2 - \frac{(m+3)(m-1)}{4}$$

$$y = x^2 - (1-m)x + 1-m$$

$$x^2 - (1-m)x + 1-m = 0$$

$$D = (1-m)^2 - 4(1-m)$$

$$= m^2 + 2m - 3$$

$$x \quad D < 0$$

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3.58

$y = x^2 - (m+2)x + 4 \rightarrow y = x^2 - (m+4)x + 1$ $y = x^2 - (m+2)x + 4 \rightarrow y = x^2 - 2x + m + 4$ ↓ $y = x^2 - (m+2)x + 4$ $y = x^2 - (m+4)x + 1$ x m m m x x $m < -6$, $m > -2$	$x^2 + x + 1 > m(x+1) \rightarrow x^2 + 2x + 5 > m(2x+2)$ $x^2 + x + 1 > m(x+1) \rightarrow x^2 + 2x + 5 < m(2x+2)$ ↓ $x^2 + x + 1 > m(x+1)$ $x^2 + x + 1 < m(x+1)$ m m $2 + x + 1 \quad m(x+1) \quad 1 \quad m$

事後テストフェーズ

3.2
3.8
3.9 3-1 3.59 60
3.9 3-2 4-1 4-2
3.11 15

3.59	3.9	3-1
3-1	$1 < k < 7$	3 2 3600

3.60	3.9	3-1
3-1	$1 < k < 7$	$-8 < k < -4$

システムの開発

開発環境

Macromedia Flash MX2004 Action Script

Perl5.8.0 Action Script

4.1

4.1

Flash	○			
Action Script		○	○	
Perl			○	○

診断機能の実装

3 5

4.2

4.2

or				
or				
	A	B		
	E	F	G	H
	I	J	K	
	M	N		

1

1

A

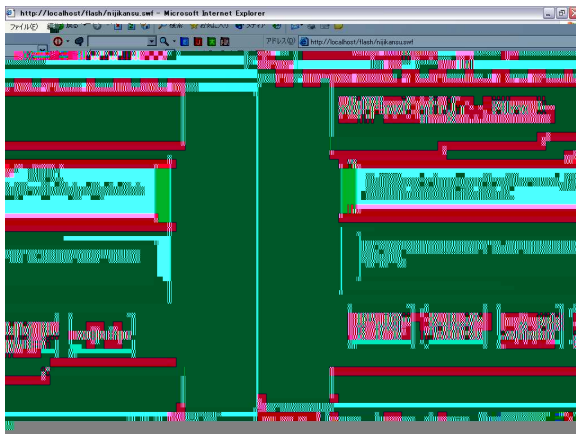
A

K

K

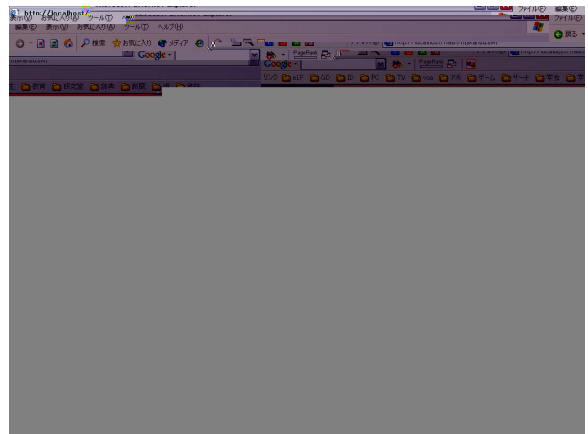
ユーザインターフェースの実装 事前テストフェーズ

4.1



4.1

4.2



4.2

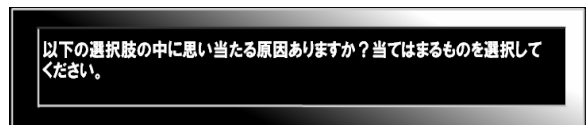
4.3

4.4

4.5

4.6

4.5



4.3

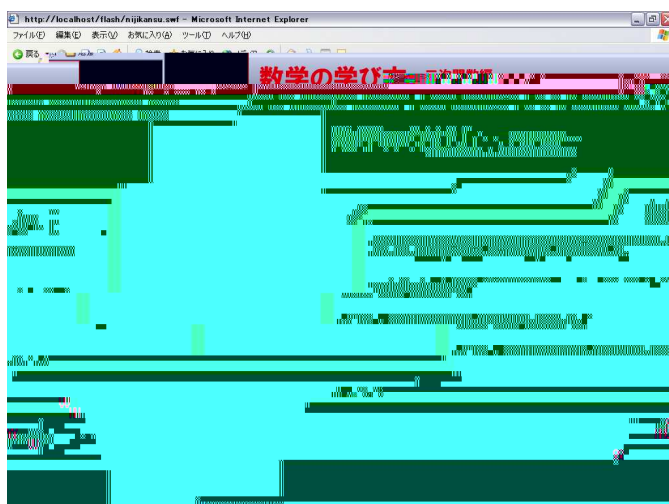
4.4



4.5

4.6

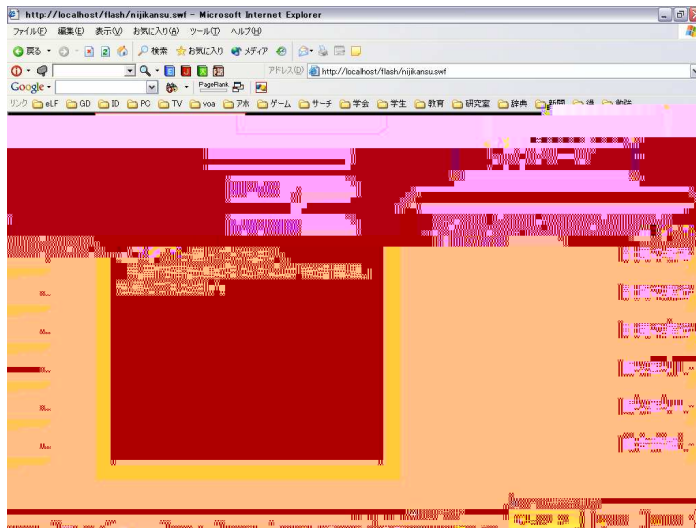
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4.7

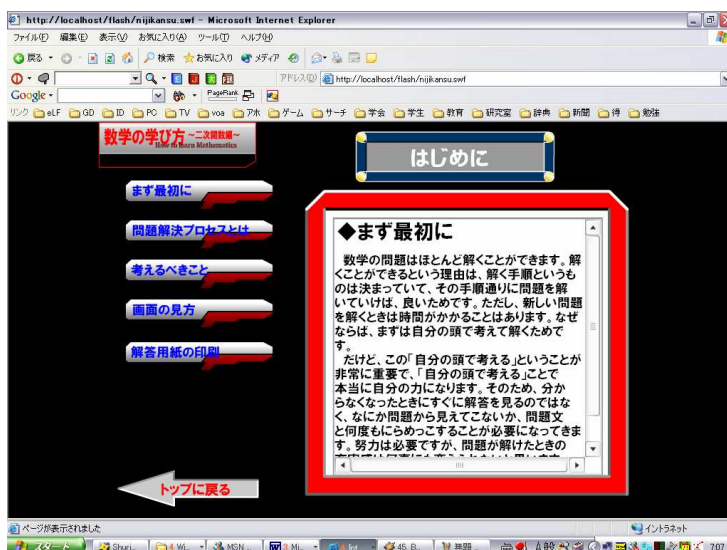
学習フェーズ

4.8



4.8

4.9

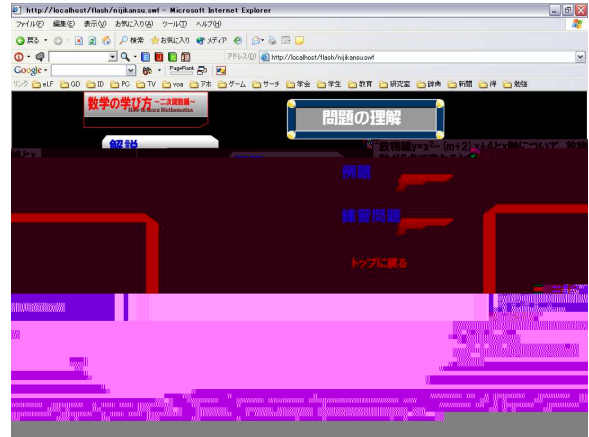
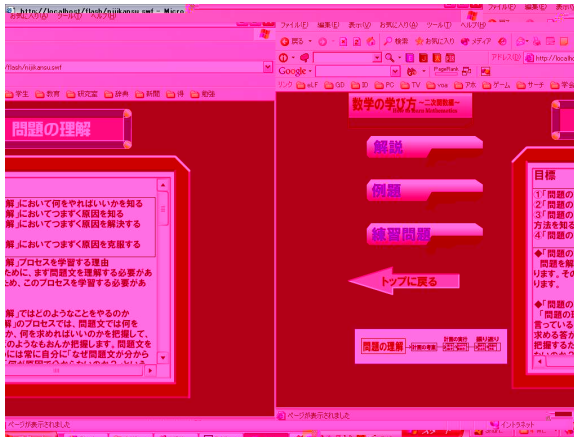


4.9

4.10

4.11

4.12



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4.11



4.12

4.13

4.14

4.15



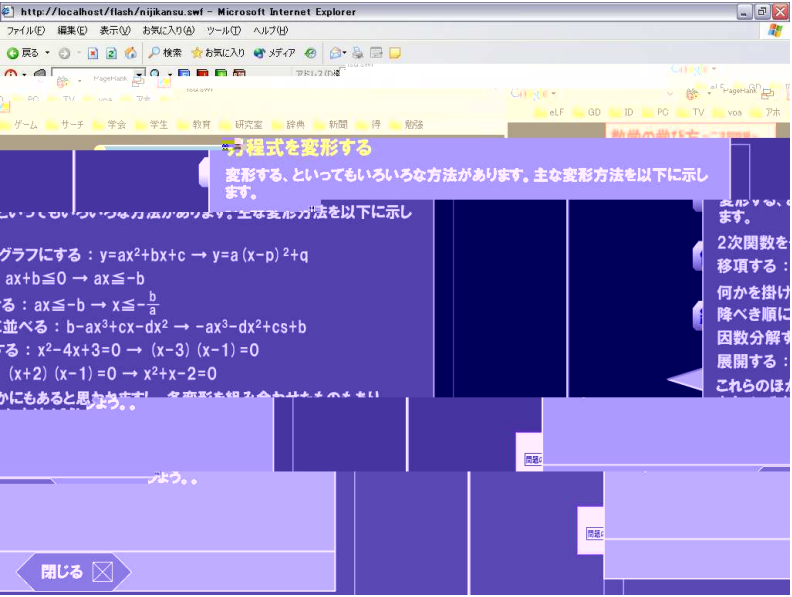
4.13

4.14



4.15

4.16



4.16

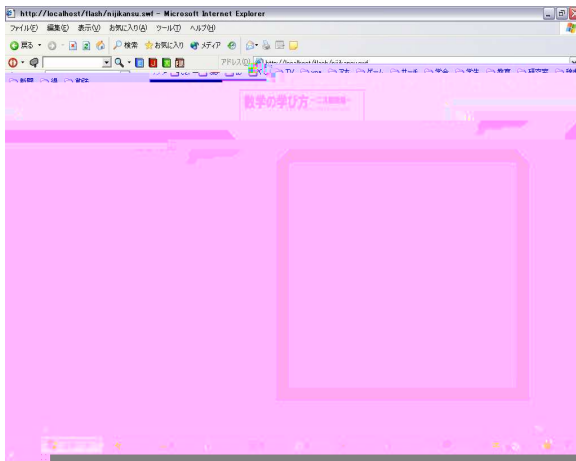
事後テストフェーズ

インターフェース制御機能の実装

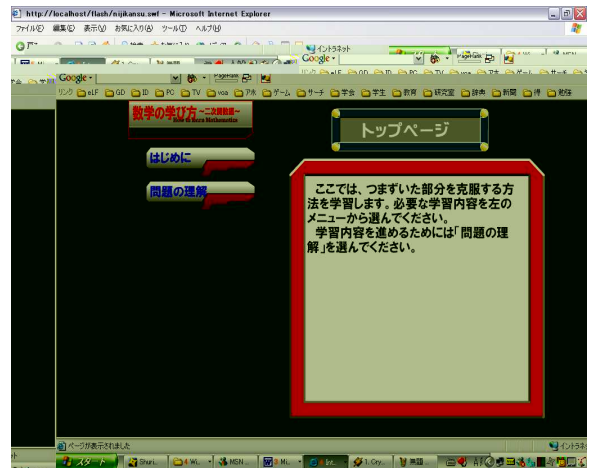
4.17

4.18

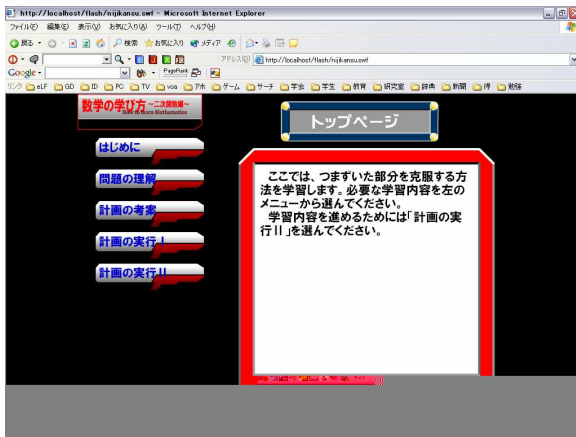
4.19



4.17



4.18



4.19

4.19

4.20

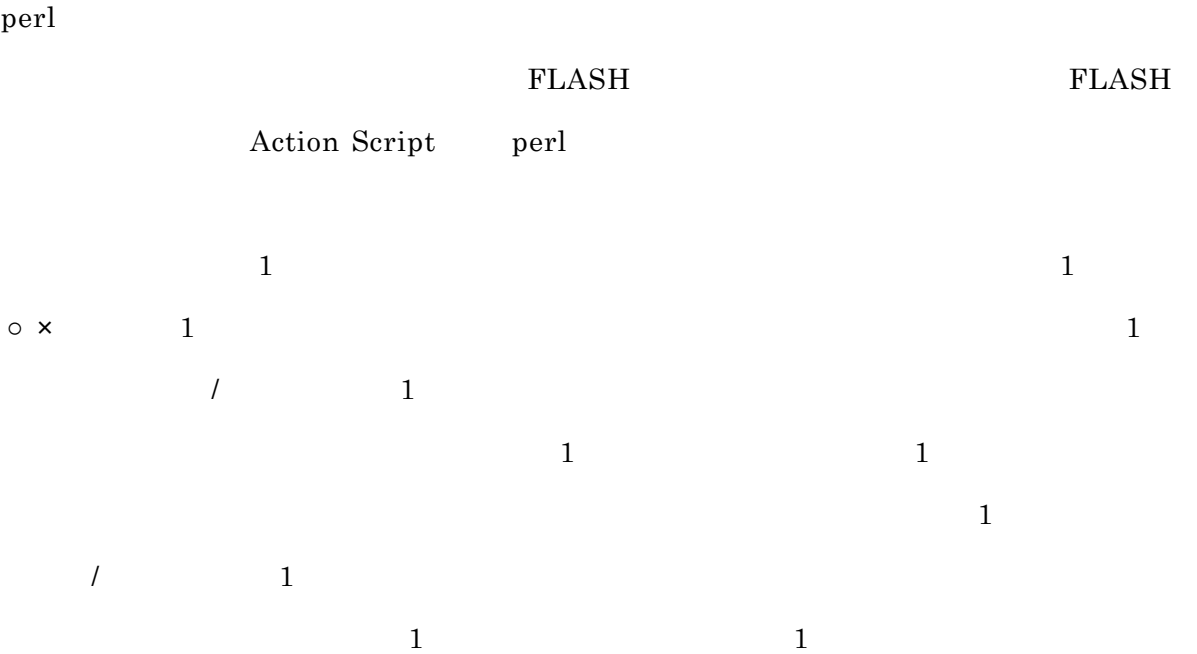


4.20

4.21

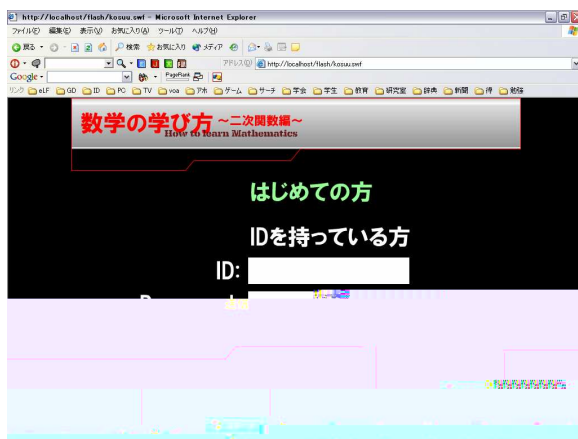
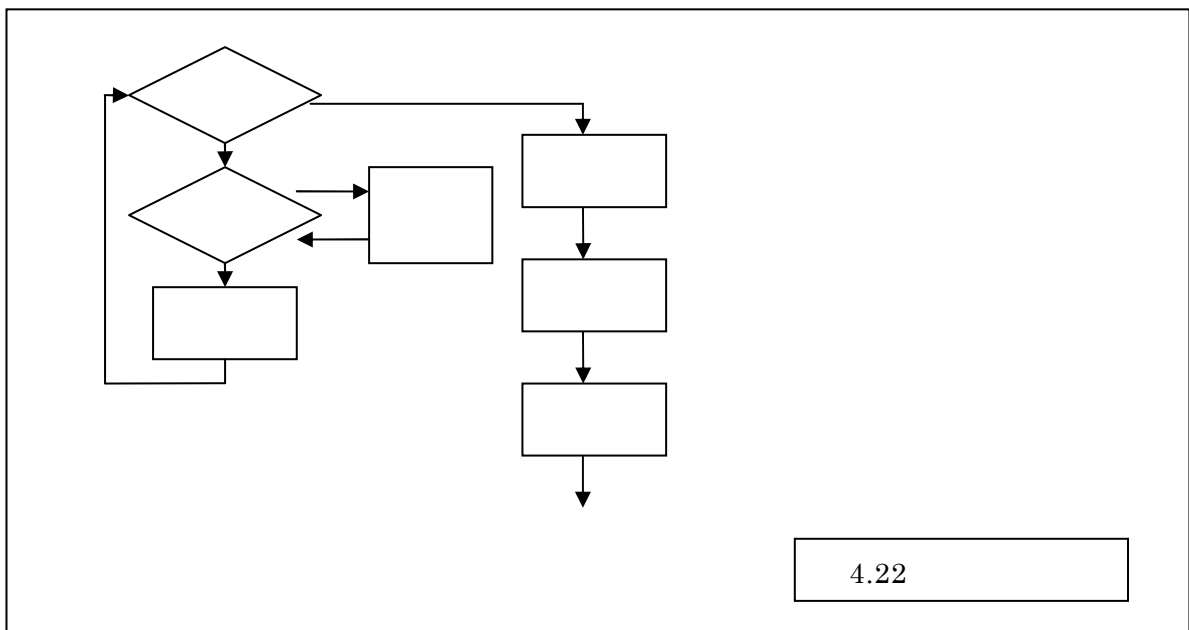
ユーザデータ収納機能の実装
事前テストフェーズ

30 4.3

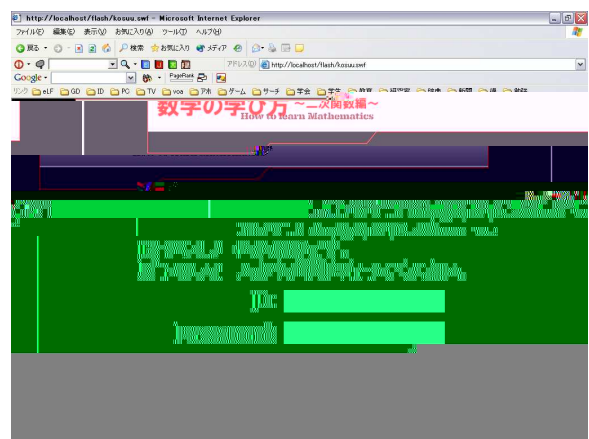


4.3

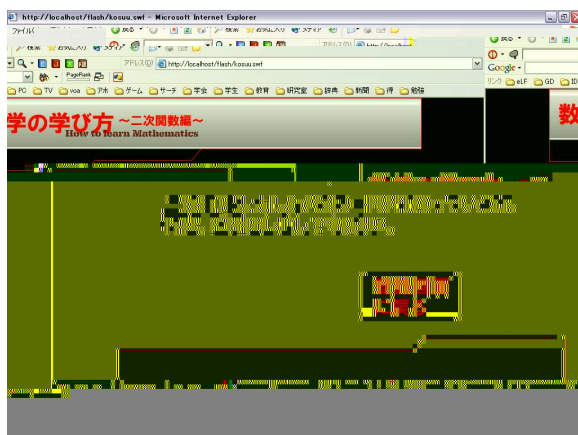
		FLASH	
\$data[0]	1 ○ ×	1pro	21/10
\$data[1]	1 /	1pro_g1	21/10
\$data[2]	1	1pro_g2	10/21-24
\$data[3]	1 /	1pro_k1	21/10
\$data[4]	1	1pro_k2	\$data[2]
\$data[5]	2 ○ ×	2pro	10-11/21-23
\$data[6]	2 /	2pro_g1	21/10
\$data[7]	2	2pro_g2	21/10
\$data[8]	2 /	2pro_k1	21/10
\$data[9]	2	2pro_k2	10/21-27
\$data[10]	3-1 ○ ×	3-1pro	21/10
\$data[11]	3-1 /	3-1pro_g1	21/10
\$data[12]	3-1	3-1pro_g2	21/10



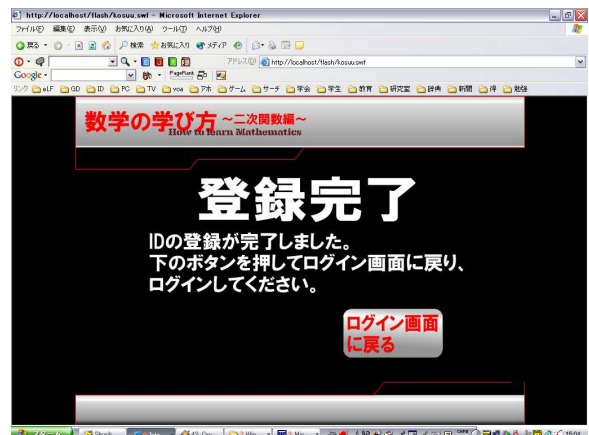
4.23



4.24



4.25



4.26

形成的評価と改善

2 次関数の 1 対 1 評価

10)

1 1

1 対 1 評価・改善(1)

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3

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2004 12 2 16 19

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0.

1.

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6.

7.

8.

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5

1.

2.

$$y=x^2-2(m-4)x+4 \quad x \quad m$$

→

$$m<-6, m>2 \rightarrow 2<m<6$$

3.

4.

5.

$$y=x^2-2(k-4)x+9 \quad x \quad 2 \quad k$$

→

$$1<k<7 \rightarrow k<1, k>7$$

$$y=x^2-2(k-4)x+9 \quad x \quad k$$

1.

2.

3.

4.

.

5.1

5.1

○	
1.	
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4.	
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8.	
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2 ○	
1.	
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3	

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5.		

1 対 1 評価・改善(2)

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2004	12	9	14	17	
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個数の処理の 1 対 1 評価

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1 対 1 評価・改善(1)

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2004 1 18 14 17

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○ ○

○ ○

7.

○ ○

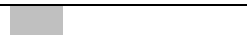
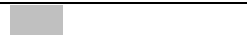
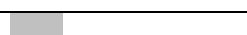
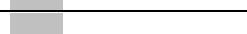
● ●

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5.3

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5.	

1 対 1 評価・改善(2)

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3

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2004 1 20 10 13

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1440 $\rightarrow$ 14400

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3

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	A		
	B		
	C		
	D		
	E		
	F		
	G		
	H		
	I		
	J		

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4

3

2

1

5.6

2.

5.6

1.	
2.	
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4.	
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6.	
7.	
8.	

5.1

10

5.2

2.5

3.3

3.5

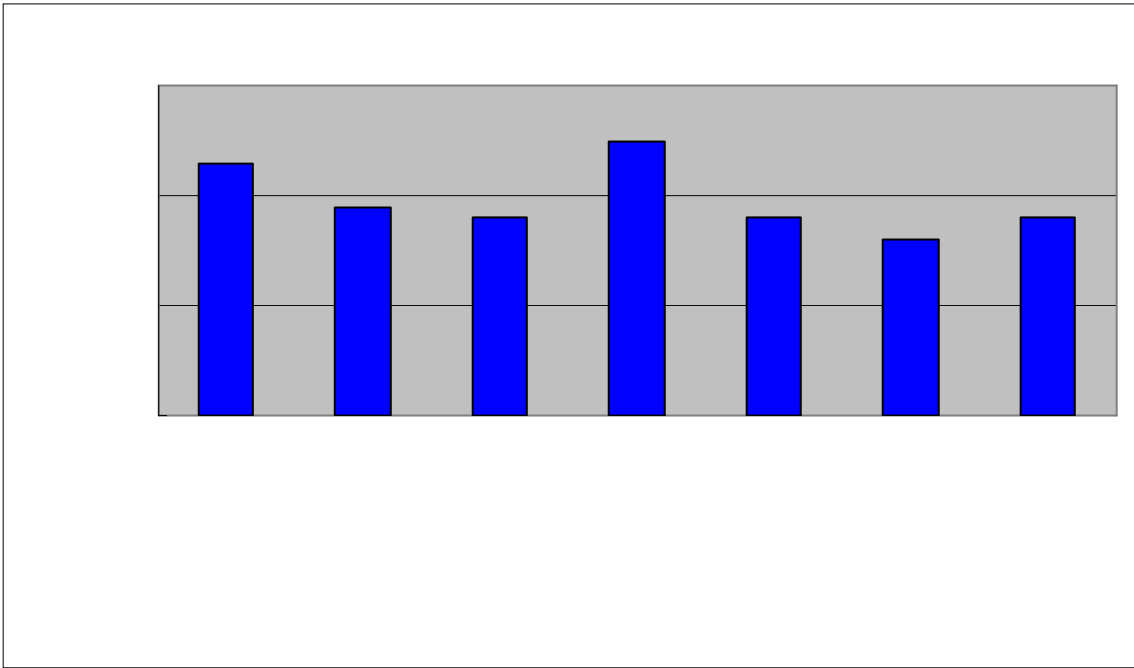
7

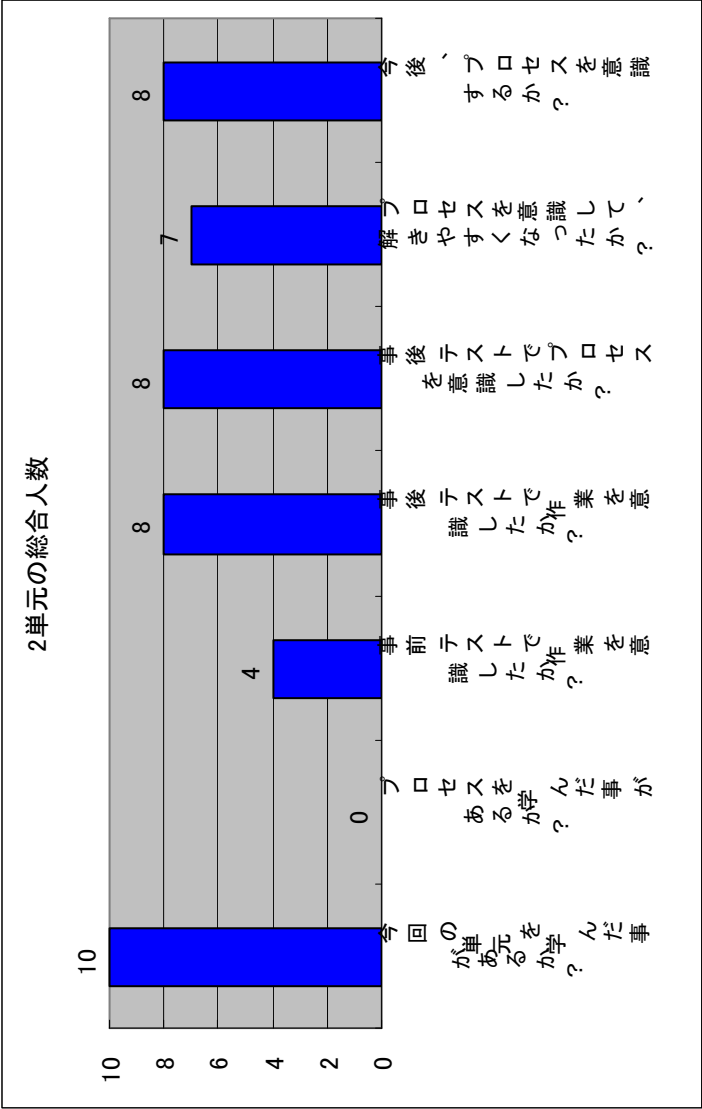
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4

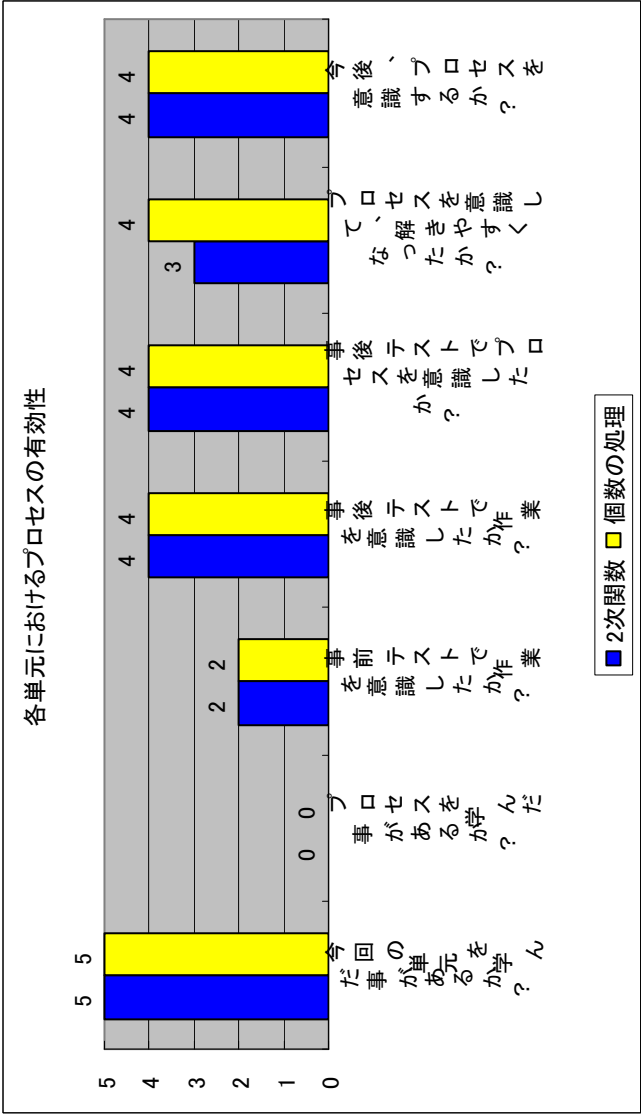
4

3.0





5.3 10



5.4

1. k m

2.

3. $3-2$

4.

5. 5 4

6.

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$\bullet$ $\circ$

A,B,C...

2 次関数の追加した問題の 1 対 1 評価

2

1 対 1 評価・改善(1)

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2004 2 4 14 17

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1.

2.

3. 1

4.1 2

5.2 1

6. 1

7.1 2

8.2 2

9. 2

10.

11.

12.

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1.

2.

- 1.
- 2.
- 3.

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5.8

5.8

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5.9

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6.	
7.	

最後に

G.

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10 7

10 7

10 8

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